Political Brinkmanship: US Debt Ceiling*

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Abstract

US debt ceiling crises are a recurrent result of political brinkmanship: we explore theoretically how such brinkmanship tactics affect gridlock, bargaining power and welfare in US budget negotiations. Failure to agree in any period implies, as in bargaining models, a status-quo disagreement payoff and a continuation of the negotiation. However, under brinkmanship, agreement failure in any period may precipitate with a small chance a debt crisis, an outcome worse than the status-quo and than any budget agreement. In equilibrium, such brinkmanship threats improve gridlock, i.e. the scope of agreement, but also increase the risk of crisis. Brinkmanship reduces welfare when one might think it is most needed: severe gridlock. In this case, despite this global welfare loss, a party has incentives to use brinkmanship strategically to obtain a favorable bargaining position.

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"Brinkmanship...the threat that leaves something to chance" (Thomas Schelling)

1 Introduction

Motivation. Political negotiations take time and present multi-faceted issues that are difficult to summarize. But one feature seems to be distinctive and recurrent in US budget politics in recent times: the strategic use of the debt ceiling. This puts 'do-or-die' conditions for an agreement to be reached. Such brinkmanship tactics provoked the US debt ceiling crises under presidents Clinton (in 1995), Obama (in 2011 and 2013) and Biden (in 2021 and 2023). For instance in 2013, the Republican Party in Congress insisted that president Obama should defund the Affordable Care Act for it to agree to raise the debt ceiling. As negotiations went on, the US Treasury took extraordinary measures (such as suspending investments on certain kinds of retirement accounts) to enable government payments and stated that payments would be delayed when the measures would not suffice anymore: the US defaulting on its debt became more likely as days passed without an agreement and would have resulted in permanent damage to the economy.[1] These brinkmanship tactics are designed to put pressure and extract concessions from the counterparts.

A similar debt ceiling episode happened in 2011. As a consequence, the U.S. government debt was downgraded for the first time in its history, leading to a significant stock market fall. Governing by crisis, engaging in fiscal brinkmanship, and threatening a nation's creditworthiness is a risky strategy. The more often the US Congress plays this game, the higher the chances of a miscalculation snowballing into a painful economic crisis. In the current highly polarized and acrimonious US Congress (McCarty [2019]), partisan discussions over the high US debt level remain salient and this risk certainly appears likely to recur.

Brinkmanship. The US sovereign debt ceiling is not expressed in terms of a percentage

[1] As Treasury Secretary Janet Yellen puts it in 2023: "It is therefore critical that Congress act in a timely manner to increase or suspend the debt limit. Failure to meet the government's obligations would cause irreparable harm to the economy, livelihoods of all Americans, and global financial stability " (Yellen [2023]).

of GDP, thus needs to *actively* be raised periodically, a rather peculiar feature unique to the US. This makes the political strategy of refusing to raise the US debt ceiling available and it is indeed used with surprising regularity. It is a brinkmanship tactic which amounts to putting a "time bomb"that can go off anytime over a negotiation. This strategy may have private or common advantages and costs. But what is key about these brinkmanship threats is that they work by creating the *risk* of an accident. Namely, they generate probabilistic outcomes that hang over the negotiation like a Damocles's sword that can fall at any moment, albeit with a small chance. Such risks were salient during the US debt ceiling crises: having substantial effects on financial markets, such brinkmanship episodes represent thus a *true risk* whose extent depends on many random factors beyond the credibility of the threatening side. As Schelling (1960) observes referring to cold war impasses: "the key to these threats is that, though one may or may not carry them out if the threatened party fails to comply, the final decision is not altogether under the threatener's control....these risks could involve chance, accident, third-party influence, imperfection in the machinery of decision, or just processes that we do not entirely understand."

Besides the peculiar context of the US debt ceiling, brinkmanship tactics are also routinely used in politics. For instance, in parliamentary regimes if an agreement is not reached, the majority dissolves with some probability and the parties in the coalition lose power. Another notable illustration comes from the negotiations between the UK and the European Union that followed the 2016 Brexit referendum (on the withdrawal agreement and then on the trade agreement). On several occasions, threats were made by the EU not to extend negotiations if an agreement was not reached before a deadline in the UK parliament, which would have resulted in a *no-deal* Brexit, possibly worse than any agreement for both the UK and the EU.[2]

[2] The US debt ceiling case is more stark though as it involves two unified parties as in our model here, whereas both the above examples are more complex and involve more than two parties in the negotiation: many parties in parliamentary regimes and many factions within the UK parliament that voted down several deals negotiated externally between the UK executive and the EU.

Trade-offs. Political brinkmanship tactics, such as putting a budget negotiation at the brink of a precipice, raise previously unstudied positive and normative questions which we aim to explore here. Namely, whether imposing such a burden upon the negotiation can be welfare improving, and in particular if a party could benefit from the small chance that a ruinous outcome, such as a financial crisis, ends the negotiation. Prima facie, there seems to be two possible benefits of do-or-die threats precipitating calamitous potential costs to all sides. One is a common benefit: making both sides willing to compromise to find an agreement more urgently thus reducing gridlock, that is reducing the cost of extended negotiations and delayed outcomes. The other is private: do-or-die threats can also be imposed strategically by a negotiating party seeking an advantageous bargaining position, for instance if the blame for a US default crisis hurts more one side than the other.[3] On the flip side, there are costs of imposing such threats if the "time bomb" happens to explode before a final agreement is reached and a US default de facto materializes, thereby hurting, possibly to a different extent, all parties.

Model. To shed light on the aforementionned trade-offs, we present a model in which two parties must repeatedly decide to approve, or not, a proposed budget agreement. These proposed budget agreements are randomly drawn every period out of a set of agreements that grant to both parties (weakly) better outcomes than the status-quo. This randomness reflects the unavoidable underlying uncertainty on what the next proposal will be if the current one fails to be approved.[4] The core setup we analyze is a standard pie-sharing framework with costly delay, namely the pie is shrinking with time. In the presence of brinkmanship however, if a proposal is rejected, then with some probability $h > 0$ (small, and zero in the benchmark case), this causes the bargaining to end with a default/crisis, namely an outcome *ruinous to both* parties. The two sides may differ in their utility from

[3]This share of blame is affected by many political factors beyond the scope of our simple model and ultimately also depends on voters' perceptions.

[4]Our focus is the final approval of a proposal previously negotiated by a committee/delegation. This negotiation may entail bargaining between several factions as well as unforeseen economic and unanticipated institutional constraints becoming binding.

this ruinous outcome, but crucially this utility is always negative, i.e., a *worse outcome than the status-quo*. Conversely, $(1-h)$ represents the chance that the ruinous outcome (i.e. the debt ceiling) is not reached, thus the negotiating process continues through one additional period in which a new proposal is drawn to be voted on, and so forth.

Small Crisis Chance. Our aim is to understand the effects of brinkmanship, namely who may benefit from such a tactic and how this affects several outcomes besides welfare, such as equilibrium gridlock (the per period chance of a deal being rejected), the equilibrium bargaining power (the expected location of an agreement) and the overall equilibrium chance of a ruinous outcome (a US debt default crisis). A necessary first step to understand the effects of brinkmanship is to disregard the origin or credibility of such threats and to take the value of h as exogenous. We abstract from modeling the intensive margin, i.e. the choice of the value of h (and asking why tactics such as not raising the debt ceiling are credible) as it necessarily implies imposing more structure on the model. A key justification for taking h as exogenous is that in practice, a ruinous outcome such as a US debt default crisis has a rather small chance of materializing. It is perceived by financial markets as a potentially catastrophic event that has a positive, albeit very low, probability of occurring. We rely heavily on the small h assumption to obtain general results and we only focus on the extensive margin in the strategic use of brinkmanship.

Results. We find that the unique stationary equilibrium is characterized by an agreement set, representing the scope for agreement, i.e. the deals acceptable by both parties. This agreement set crucially also represents an (inverse) measure of political *gridlock*. We show that, regardless of how ruinous the default may be for each side, a larger brinkmanship threat h enlarges the equilibrium scope for agreement, making an agreement more likely in every period. Indeed, a higher h always succeeds in easing gridlock by forcing parties to compromise more overall (although one party may compromise less to extract a bargaining advantage). While this could have been anticipated, the effects on welfare however are subtle.

The overarching trade-off is that, while increasing brinkmanship h is effective in alleviating gridlock, it is also more dangerous: the default may become more likely in equilibrium, which in turn reduces expected welfare. In particular, we show that regardless of the parameters and the size of brinkmanship threats, welfare only depends on one endogenous statistic, *gridlock severity*, albeit in a non-trivial way. Namely, brinkmanship helps total welfare only when gridlock is not severe. Interestingly, if we start from bargaining positions of severe gridlock (in which disagreement is more likely than not) then enacting a brinkmanship tactic, while easing gridlock, hurts total welfare as the risk of a ruinous outcome outweighs the agreement benefits. Despite ruinous welfare consequences overall in this case, a party can have private benefits from brinkmanship: if one party is advantaged in that it has a lower (perceived political) cost of the potential default/crisis, then enacting political brinkmanship shifts the whole agreement set to its advantage. This happens independently from the size of the advantage.

Summary. Our model allows us to study the effects of brinkmanship tactics in situations of more or less severe gridlock and proposes a rationale for them. Four main insights emerge. First, brinkmanship is always effective in alleviating gridlock and promoting compromise overall. Second, it hurts total welfare precisely in situations of severe gridlock, and thus cannot provide a good solution for gridlock. Third, in situations of severe gridlock, despite the welfare loss overall, brinkmanship can still be promoted by an advantaged party, which has, or perceives to have, a lower cost in case of a financial default. Fourth, in situations of mild gridlock, brinkmanship is welfare improving overall and can be promoted by either party if neither is too disadvantaged.

In the following, after the literature review, we introduce the general model and its results. To keep the body of the paper compact all proofs are relegated to the appendix.

2 Related Literature

To the best of our knowledge no theoretical papers have so far tried to analyze explicitly the positive and normative implications of the US debt ceiling political brinkmanship tactics. This paper touches on several other strands of related literature which we outline below.

War Brinkmanship. Powell [1988] looks at "nuclear brinkmanship" in a game of escalation, where the risk of breakdown is endogenously determined and the outcome is binary: either one side or the other wins. Our model is more akin to a bargaining model (since the share of the pie obtained by each party is partly endogenous, taken in a large set of possibilities), as it looks at how brinkmanship changes a continuous bargaining outcome rather than a bang-bang conflict outcome, both in terms of pie split and in term of delay. Schwarz and Sonin [2008] consider brinkmanship in a dynamic setting where a potential aggressor demands concessions from a weaker party under the threat of war[5] and prove that a continuous stream of transfers prevents war more effectively than a lump-sum transfer in the absence of commitment. Acharya and Grillo [2015] study a model of war and peace in which leaders believe there might be crazy types who always behave aggressively regardless and escalate to a war from a peaceful settlement. This "madman" brinkmanship strategy can be used by non-crazy types to their advantage, as they show. As in our model, their outcome is ruinous (war) with positive probability. Though we do not model this signalling game, one interpretation of our model has a similar underpinning, as under brinkmanship a debt default crisis also materializes with positive (albeit small) probability.

Political Brinkmanship. Some scholars have studied forms of costly political brinkmanship in the US context. Patty [2016], motivated by US government shutdown episodes, proposes a model in which obstruction in a legislative process is unambiguously costly to all parties due to its consequent delay and gridlock, but incumbents might decide to use this brinkmanship as a signalling device, a show of strength and resolve vis-à-vis voters. Grillo

[5] At each period of their model, the aggressor decides whether to start war whereas the weaker party decides the transfer to the aggressor.

and Prato [2020] propose a theory of democratic backsliding which, as they show, may occur even if citizens intrinsically value democracy. Backsliding, an attack on democratic institutions, can be interpreted as institutional brinkmanship. In a similar spirit to ours, they show how opportunistic authoritarian governments may want to attack democratic institutions to gain an advantage.

Collective search. Our modeling strategy borrows from the collective search and experimentation models, in which a group chooses every period between accepting the current negotiation outcome or waiting for a new outcome next period[6]. For instance, Compte and Jehiel [2010] show that more stringent majority requirements select more efficient proposals but take more time to do so. Albrecht et al. [2010] find that committees are more permissive than a single decision maker facing an otherwise identical search problem.[7] Compte and Jehiel [2017] push further the same approach for large committees characterizing the optimal majority rule. Similarly, several authors assume different types of stochasticity. Strulovici [2010] and Messner and Polborn [2012] focus on committee decisions in which preferences are unknown and only learned over time, thus the option to delay happens in equilibrium albeit with different degrees of efficiency depending on the majority rule. Ortner [2017] analyzes how shocks to the popularity of politicians affect bargaining outcomes. Moldovanu and Rosar [2021] study voting in a Brexit-like model with one irreversible option and compare the effect of different voting rules. They show that voting by supermajority over two consecutive periods dominates voting by simple majority. Basak and Deb [2020] focus on a bargaining environment in which public opinion shocks provide leverage by making compromises costly in the presence of deadlocks.

Stochastic bargaining. In our model offers/deals are exogenously stochastically drawn

[6]This literature is somehow related to a classic literature on bargaining where both parties are allowed to search for outside options, see Wolinsky [1987] and Chikte and Deshmukh [1987] for classic treatments on the question. See Muthoo [1995] that analyzes the role of parties being able to leave temporarily the negotiation and Manzini and Mariotti [2004] where bilateral bargaining occurs between parties that can agree on a joint outside option.

[7]In a related model with interdependent values, Moldovanu and Shi [2013] study costly search for a committee and study how acceptance thresholds and welfare depend on the degree of conflict within the committee.

from the set of mutually beneficial outcomes to both parties, this generates naturally inefficient delays in finding agreements. There is a vast literature of legislative bargaining models with endogenous offers in which elements of stochasticity generate inefficient delays in agreements or gridlock in the presence of an endogenous status-quo. Several papers analyze stochastically evolving preferences, see Dziuda and Loeper [2016] or Bowen et al. [2017]. Other works explore the case of delay with a stochastic total surplus, such as Merlo and Wilson [1995], Merlo and Wilson [1998] and Eraslan and Merlo [2002].

Bargaining with deadlines. Several authors have looked at the effect of hard deadlines in negotiations, which nicely complements our stationary setup. In other words, while we study dynamic negotiation between two parties in the presence of a stationary stochastically extendable deadline, in most of the literature, the deadline is tight in the sense that no extension is possible.[8] This generates incentives to reach agreements in the "eleventh hour", that is at or very close to the deadline (see Simsek and Yildiz [2016] for the role of optimism in these models). Such (non-stationary) games have been studied by Ma and Manove [1993], Fuchs and Skrzypacz [2013] and others[9].

3 Model

3.1 Setup

There are two parties, denoted by $\theta = 0$ and $\theta = 1$. Each party $\theta \in \{0, 1\}$ has single-peaked preferences over $X = [-1, 1]$, the set of all possible budget *deals*. Specifically, we assume that $u_0(x) = 1 - x$ and $u_1(x) = 1 + x$, so that party 0 (resp. 1) prefers lower (resp. higher) deals.

The final outcome may be a deal in X or the *ruinous outcome* d^*, which henceforth

[8] See also Ellingsen and Miettinen [2008] who study a model of bilateral bargaining where negotiators can write binding contracts and show that conflict is frequently the unique equilibrium outcome when commitments technologies are highly credible.

[9] See Cramton and Tracy [1992] for empirical evidence or Gneezy et al. [2003] for experimental evidence on this observation.

will denote a US default crisis and its consequences. The option d^* does not lie in X and yields each party $\theta \in \{0,1\}$ a utility $d_\theta < 0$. We denote by $D = d_0 + d_1 < 0$ the ruinous outcome's overall *severity*, and by $B = d_1 - d_0$ party 1's *advantage* in terms of ruinous outcome payoffs relative to party 0. This advantage could be actual or simply perceived: e.g. represent a perceived political advantage based on which side voters would blame more if a crisis materializes. Without loss of generality, we assume that $B \geq 0$, and if $B > 0$, we refer to 1 as the *advantaged* party and to 0 as the *disadvantaged* one. The main parameters of the model are represented on Figure 1.

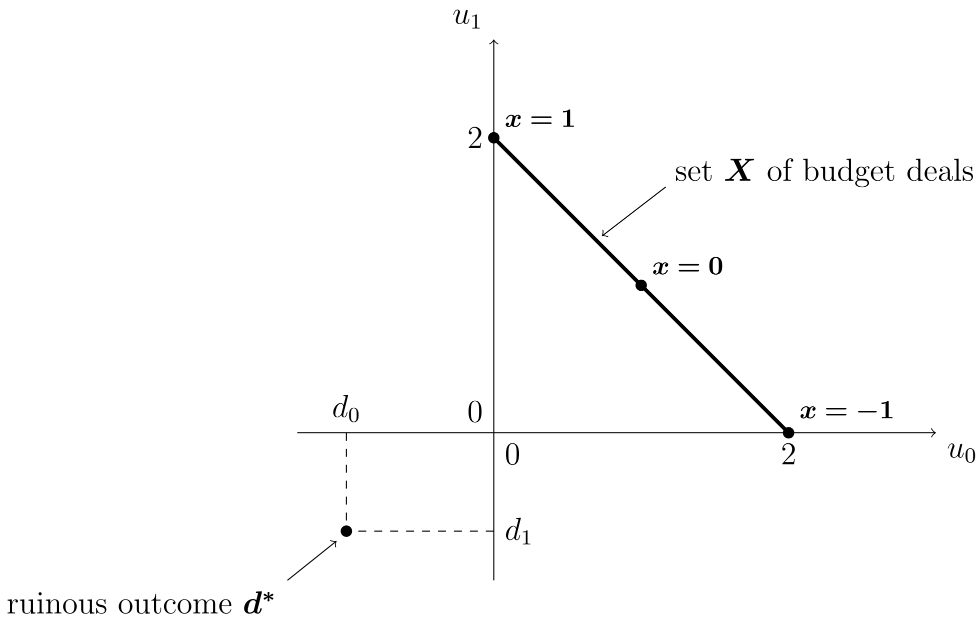

Figure 1: Utilities in the Bargaining Model

In order to focus on the effects of the ruinous outcome payoffs on delay/gridlock and welfare, we assume a simple bargaining protocol which naturally allows for some stochastic delay in equilibrium. The bargaining procedure is sequential and the agenda has a stochastic aspect: for each $t \in \mathbb{N}$, a proposed deal x_t is drawn, from the set of budget deals X, uniformly and independently from previous draws.[10] Then, parties simultaneously choose to accept or

[10]The reduced-form assumption that proposals are randomly generated provides a tractable way to account for the inevitable uncertainty surrounding future proposals. This assumption is common in the bargaining literature, see for instance Penn [2009], Compte and Jehiel [2010] and Acharya and Ortner [2022]. The former article provides a detailed discussion of this assumption.

reject the proposal x_t. If both accept it at period t, the final budget outcome is x_t. Otherwise, the resulting outcome is governed by a binary outcome H_t which takes value 1 with a small probability h, and value 0 with probability $1-h$. If $H_t = 1$, budget negotiations delays precipitate a US debt default crisis: a ruinous outcome d^* is obtained at period t. If, on the contrary, $H_t = 0$, then in that period the crisis is averted and budget negotiations continue with both parties moving to the next period of negotiation $t+1$. The parameter $h \in [0,1)$ thus represents the *brinkmanship level*, the chance the ruinous outcome materializes in any period in which budget disagreement persists.[11] In the benchmark case there is no brinkmanship, thus $h = 0$. Even when brinkmanship is present, we will assume that h is small. This is the interesting and empirically relevant case as the risk of debt crisis on any day during the US debt ceiling brinkmanship episodes was positive (and felt by financial markets), but small.[12] Both parties share the same discount factor $\beta \in (0,1)$.

3.2 Stationary Equilibrium

The strategy of each party consists in accepting or rejecting budget deals as they arrive. We restrict our attention to stationary strategies. For a given (stationary) strategy profile, we denote by $A \subseteq X$ its *agreement set*, i.e. the set of budget deals that would be accepted by both sides if proposed. We denote by w_θ party θ's *reservation value*, i.e. her expected continuation utility when she rejects a deal. By stationarity, w_θ satisfies the following recursive equation:

$$w_\theta = \overbrace{hd_\theta}^{\text{ruinous outcome } d^*} + \overbrace{\beta(1-h)\mathbb{P}(x \in A)\mathbb{E}[u_\theta(x) \mid x \in A]}^{x \text{ lies in the agreement set}} + \overbrace{\beta(1-h)\mathbb{P}(x \notin A)w_\theta}^{x \text{ does not lie in the agreement set}}.$$

The previous formula can be read as follows : when a deal is rejected, the ruinous outcome

[11] Such an exogenous risk of breakdown also appears in one of Binmore et al. [1986]'s approaches to relate strategic bargaining models to the Nash bargaining solution.

[12] In an earlier version of the paper, we solved the model for any level of h (results are available upon request). Mathematically, β and h enter in a similar way in the equations. The key difference is the ruinous outcome term hd_θ (and the fact that we only study extensive margins for brinkmanship: h is either zero or small).

arises with probability h; with complementary probability $(1-h)$ the game continues to the next stage, where payoffs are discounted by β; in that stage, either the proposal x is accepted and the expected payoff is that of an average deal in the agreement set A, or the proposal x is rejected, in which case the payoff is the reservation value w_θ.

We assume that each party accepts a budget deal if and only if her payoff is greater than or equal to her reservation value w_θ, even if her opponent rejects it (thus making her a priori indifferent between accepting and rejecting).[13] Then, the condition for a stationary profile with reservation values (w_0, w_1) to be an equilibrium is that its agreement set satisfies $A = A_w = \{x \in X \mid u_\theta(x) \geq w_\theta,\ \forall \theta \in \{0,1\}\}$. Hence, this profile is an equilibrium if and only if

$$w_\theta = \frac{hd_\theta + \beta(1-h)\mathbb{P}(x \in A_w)\mathbb{E}[u_\theta(x) \mid x \in A_w]}{1 - \beta(1-h)\mathbb{P}(x \notin A_w)}. \tag{1}$$

The key (endogenous) outcome variables of our analysis are the following. We denote by $\lambda = \frac{|A|}{|X|} = \frac{|A|}{2}$, where $|\cdot|$ denotes the Lebesgue measure, the *agreement probability* in every period. The variable $(1-\lambda)$ also represents the level of equilibrium *gridlock* in the legislature. We denote by c the *center* of the agreement set A, which is a measure of the relative bargaining strength of each side in equilibrium: the farther c is from zero the stronger is the equilibrium bargaining power of one party relative to the other. With these notations, the agreement set can be written as $A = [c-\lambda, c+\lambda]$, as illustrated on Figure 2 below.

[13] This assumption is a mild refinement of rationality and has a similar flavor to the one of partial honesty used in mechanism design (see [Dutta and Sen, 2012] among others). The role of this assumption is simply to rule out equilibria where both parties simultaneously reject a deal that they both strictly prefer to the outcome they obtain in case of rejection.

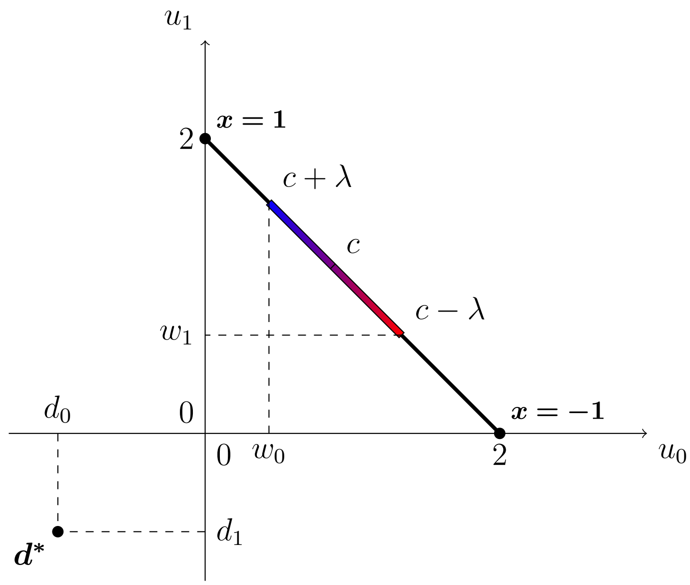

Figure 2: Agreement set in the Bargaining Model

4 Gridlock and Joint Welfare

4.1 Agreement set

In this section, we fully characterize the (unique) equilibrium agreement set when brinkmanship h is small enough, as specified below. In this case the equilibrium is *interior*, namely its agreement set is strictly contained in the proposal set X, or equivalently both agents have positive reservation values, i.e. $w_0, w_1 > 0$.

Theorem 1 *There exists $\overline{h} > 0$ such that for any $h < \overline{h}$, there exists a unique stationary equilibrium and this equilibrium is interior.*

Henceforth, we restrict our analysis to $h < \overline{h}$, i.e. to the cases where h is either zero (benchmark case of no brinkmanship) or small enough when positive.

Proposition 1 *The agreement probability λ is increasing in the brinkmanship level h. If party 1's advantage B is positive, then the relative bargaining power c is increasing in the brinkmanship level h.*

To illustrate Proposition 1, we display on Figure 3 the upper and lower bounds and relative bargaining power i.e. the center c of the agreement set as functions of the threat h, in a numerical example. The blue (resp. red) curve represents the worst deal that the disadvantaged (resp. advantaged) party is willing to accept, which coincides with the upper bound (resp. lower bound) of the agreement set.

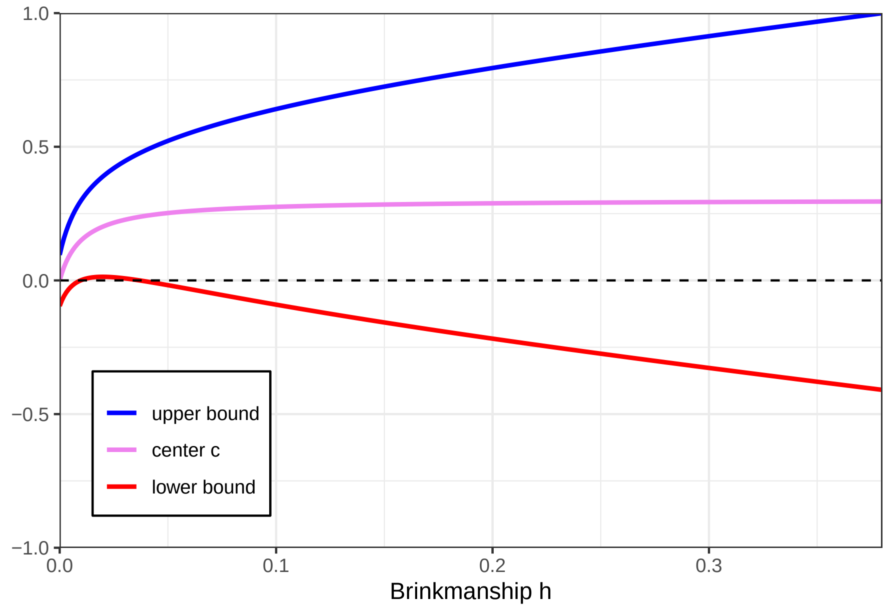

Figure 3: Agreement set for $\beta = 0.99$, $D = -1$ and $B = 0.6$.

The first key point from Proposition 1 is that *brinkmanship eases gridlock*: the scope for agreement, or overall willingness of both parties to accommodate the other party, as measured by the agreement probability λ, always increases with the brinkmanship level h. Note that, although parties compromise more overall under brinkmanship, it could be that one party compromises less. This is indeed what happens on Figure 3 for low values of h, as party 1 compromises less to push her bargaining advantage.

Second, the relative bargaining power in equilibrium represented by the center of the agreement set c, namely the budget deal outcome obtained on average, exhibits a monotonic pattern. Starting from the benchmark of no brinkmanship (i.e. no binding debt ceiling), c increases when brinkmanship is turned on, making the advantaged party better-off as long as a budget is agreed upon before a debt default crisis ensues.

We now describe how, for a given level of brinkmanship h, the characteristics of the ruinous outcome (its severity D and asymmetry B) affect the bargaining power and gridlock.

Proposition 2 *The relative bargaining power c and agreement probability λ are such that*

$$c = \frac{\Phi B}{2} \qquad \text{and} \qquad \lambda = \frac{1}{2\Delta}\left(\sqrt{1 + 4\Delta\left(1 - \frac{\Phi D}{2}\right)} - 1\right),$$

where $\Phi > 0$ and $\Delta > 0$ only depend on h.

Firstly, for a given brinkmanship level, the equilibrium bargaining advantage c only depends on the payoff advantage in case of ruinous outcome B. In particular, conditional on a budget agreement, a budget deal is closer to party 1's bliss point when this advantage is larger. Second, the level of gridlock or the agreement probability λ only depends on the severity D: when the ruinous outcome is more severe ($|D|$ is higher), parties are more likely to compromise overall and gridlock is less severe.

4.2 Joint Welfare

We now characterize parties' equilibrium joint welfare, to try to understand if and when brinkmanship is ever beneficial overall. Denoting by W_θ party θ's expected welfare, our key outcome variable here is $W = \frac{W_0 + W_1}{2}$ the *joint welfare*, which proxies the total average benefit from the expected bargaining outcome. Welfare W_θ satisfies the following recursive equation:

$$W_\theta = \overbrace{\mathbb{P}(x \in A)\mathbb{E}[u_\theta(x) \mid x \in A]}^{x \text{ lies in the agreement set}} + \overbrace{\mathbb{P}(x \notin A)\left(hd_\theta + \beta(1-h)W_\theta\right)}^{x \text{ does not lie in the agreement set}}.$$

The welfare W_θ is determined at the beginning of a period. Thus, either the randomly selected deal x belongs to the agreement set, in which case it yields an expected utility $\mathbb{E}[u_\theta(x) \mid x \in A]$, or it fails to do so and hence, either the ruinous outcome is selected or a new period starts, with expected welfare W_θ. Therefore, party θ's welfare is given by:

$$W_\theta = \frac{(1-\lambda)hd_\theta + \lambda\mathbb{E}[u_\theta(x) \mid x \in A]}{1-\beta(1-h)(1-\lambda)}. \quad (2)$$

The next result characterizes parties' joint welfare.

Proposition 3 *Parties' joint welfare at equilibrium solely depends on the agreement probability λ, and is given by:*

$$W = 1 - \lambda + \lambda^2.$$

Proposition 3 shows that parties' welfare is a non-monotonic (convex) function of equilibrium agreement probability, the endogenous variable λ, and it is symmetric around $\lambda = 1/2$. But since, in turn, λ is increasing in h, it follows that welfare is a *non-monotonic* function of the brinkmanship level h. Thus, brinkmanship has very different effects depending on the severity of gridlock. In particular, when gridlock is not severe, that is when the agreement probability λ is above 1/2, then brinkmanship is desirable as it increases welfare. But, more interestingly, the converse can also be true, that is:

Corollary 1 *When gridlock is severe (disagreement is more likely than not, i.e. $\lambda \leq 1/2$), then parties' joint welfare is decreasing in brinkmanship h.*

The case above is particularly insightful as it states that political brinkmanship hurts precisely when it is most needed to break the gridlock. Put differently, when disagreement is large to begin with, allowing for political brinkmanship is bad for welfare overall because the negative effect of a higher likelihood of default crisis outweighs the gain from the enlarged agreement scope. In sum, it is too risky to use brinkmanship when gridlock is severe.

4.3 Intuition for Welfare Result

Here we shed light on the forces which govern the link between welfare and brinkmanship. We decompose parties' joint welfare as the product of two factors: *instant welfare* and *timing factor*.

Proposition 4 *Parties' joint welfare can be expressed as:*

$$W = \underbrace{\left(1 - \mathbb{P}(d^* \mid A) + \left(\frac{D}{2}\right)\mathbb{P}(d^* \mid A)\right)}_{\textit{instant welfare}} \times \underbrace{f\Big(h\Big)}_{\textit{timing factor}},$$

where the timing factor $f(h)$ is increasing in h.

The instant welfare is the undiscounted welfare of the *eventual* budget bargaining outcome. As the average utility of an accepted bargaining deal is 1, the instant welfare is a convex combination of 1 and $\frac{D}{2}$, weighted by the overall risk of default $\mathbb{P}(d^* \mid A)$. The timing factor only depends on brinkmanship h and accounts for the *expected delay* incurred in reaching an agreement through the bargaining process.

The shape of welfare as a function of brinkmanship h thus depends on the relative importance of those two terms. The timing factor is increasing in brinkmanship h, as parties reach an agreement sooner with brinkmanship. By contrast, the instant welfare decreases with brinkmanship, as the overall risk of default $\mathbb{P}(d^* \mid A)$ is null for $h = 0$ but becomes positive for $h > 0$. To provide intuition on Corollary 1, we may write:

$$\left.\frac{\partial W}{\partial h}\right|_{h=0} = f'(0) + f(0)\left(-1 + \frac{D}{2}\right)\left.\frac{\partial \mathbb{P}(d^* \mid A)}{\partial h}\right|_{h=0} = f'(0) + f(0)\left(-1 + \frac{D}{2}\right)(1 - \lambda).$$

We see here that the positive effect of brinkmanship on welfare (through the timing factor) is constant, while its negative effect (through the risk of default, or equivalently the instant welfare) increases with the gridlock level $(1 - \lambda)$. Hence, brinkmanship becomes welfare decreasing when gridlock is too severe.

5 Strategic Use of Brinkmanship

In the previous section we analyzed the effects of political brinkmanship and explored when brinkmanship can be jointly welfare improving. In this section, we take a step back and focus on whether political brinkmanship may benefit one particular party (compared to the benchmark of no brinkmanship), and may thus be used strategically to gain a bargaining advantage by one or either of the two parties. As before, we consider the empirically relevant case in which the brinkmanship threat is small and positive.

5.1 Brinkmanship by Advantaged Party

We focus first on the advantaged side, party 1. When, absent brinkmanship, gridlock is mild ($\lambda > 1/2$), we know from Proposition 3 that brinkmanship is welfare-improving. Clearly, party 1 has an incentive to use brinkmanship strategically. The following result provides sufficient conditions under which party 1 will be willing to use brinkmanship strategically (i.e. not raise the US debt ceiling), even when it is welfare-decreasing overall.

Theorem 2 *Party* 1 *has incentives to use of brinkmanship in each of the following cases:*

1. *for any advantage $B > 0$, if gridlock is severe enough (λ small enough)*
2. *for any (sufficiently severe) ruinous outcome $D < -1/8$, if the advantage B is high enough.*

Theorem 2 emphasizes that there are several grounds for the strategic use of brinkmanship by party 1 even though it decreases welfare overall. One sufficient condition is that initial gridlock be high enough, and it holds for any severity and any asymmetry of the ruinous outcome. Indeed, when gridlock is high, the private bargaining advantage obtained by party 1 (location of an expected budget deal, i.e. c) is an order of magnitude higher than the common drop in welfare. The second condition states that whenever the ruinous outcome's severity is not negligible ($D < -1/8$), enough asymmetry between the two sides in ruinous outcome payoffs ensures that the advantaged party profits from engaging in brinkmanship tactics. The reason for this is that the bargaining concessions made by the disadvantaged party 0 in equilibrium are sufficient to overturn the drop in welfare.

The welfare consequences of the brinkmanship strategy are illustrated in Figure 4.

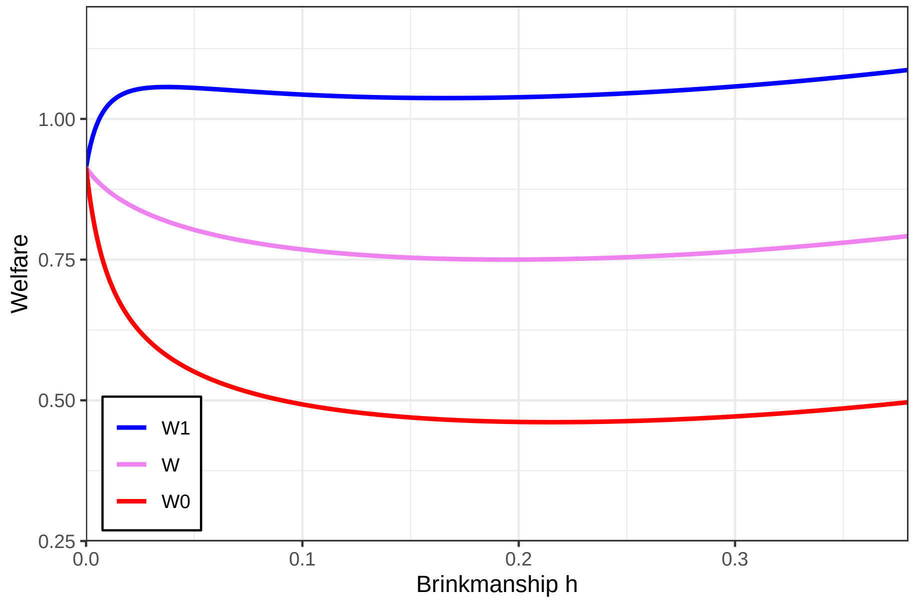

Figure 4: Parties' welfare for $\beta = 0.99$, $D = -1$ and $B = 0.6$

In the example depicted in Figure 4, absent brinkmanship gridlock is severe: the agreement probability in any given period is $\lambda \approx 10\%$. Hence, a small brinkmanship threat is welfare-decreasing overall, but the brinkmanship strategy is nevertheless advantageous for party 1.

5.2 Brinkmanship by Either Party

The brinkmanship strategy is never profitable for Party 0 in situations of intense gridlock ($\lambda \leq 1/2$) as illustrated on Figure 4, as it only worsens its welfare. Yet, in some situations which we outline below even the disadvantaged party 0 may want to use brinkmanship.

Proposition 5 *Party* 0 *also has incentives to use brinkmanship when gridlock is mild* ($\lambda > 1/2$) *and its disadvantage* B *is small enough.*

The intuition for Proposition 5 is that absent significant gridlock, brinkmanship improves the willingness to compromise of parties so much that even the disadvantaged party may benefit from it. We illustrate on Figure 5 below that the positive welfare consequences for the disadvantaged party only accrue when the disadvantage is small enough.[14] Note that in this case, the brinkmanship strategy is Pareto-improving.

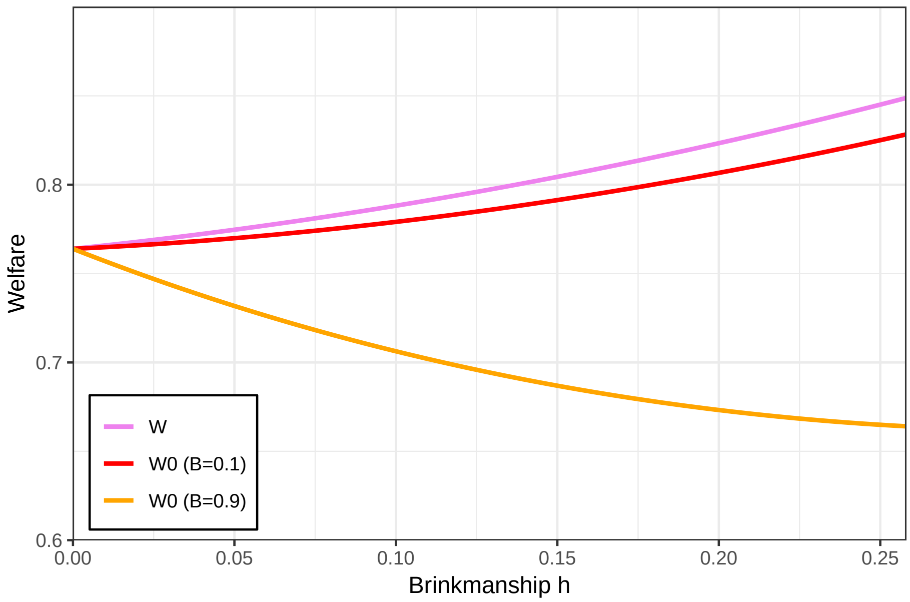

Figure 5: Joint and party 0's welfare for $\beta = 0.5$ and $D = -1$

6 Conclusion

The broad question of how political brinkmanship tactics during a budget negotiation may alter outcomes is key and recurrent in US budget politics. This question is theoretically still largely unexplored, thus given its salience we believe further research is needed on this issue. In this paper we analyzed how the use of brinkmanship such as the refusal to raise the debt ceiling can affect a budget negotiation, in a simple model which features stochastic agreement delay. In particular, we studied how brinkmanship affects gridlock and delay, the

[14]Note that the joint welfare W does not depend on B, by application of Proposition 3 (W only depends on λ) and Proposition 2 (λ does not depend on B).

welfare of the negotiating parties and finally when parties have incentives to use of such brinkmanship tactics for expected political gains.

The key takeaway is that in situations of severe gridlock political brinkmanship hurts global welfare of the negotiating parties as it raises the risk of financial crisis too much. However, even in these situations of gridlock one party has incentives to use this brinkmanship tactic to gain a bargaining advantage in the negotiation. In sum, our theoretical framework allows us to disentangle shortcomings but also advantages of the debt ceiling, an institutional anomaly peculiar to the US system, and illustrates precisely when the statutory debt limit can be exploited strategically by one party while hurting the negotiation overall.

Our model is extendable to more general distributions of potential agreements and to asymmetric information in which the payoffs to ruinous outcomes are private information of the two sides. Finally, we stayed away from credibility issues assuming small brinkmanship which seems empirically relevant. But the results of our model may be used as the last stage of a broader model which endogenizes the credibility of walk-away do-or-die threat announcements.

A Proofs

We introduce two parameters that will be used throughout the proofs. We note $\Phi = \frac{h}{1-\beta(1-h)}$ and $\Delta = \frac{\beta(1-h)}{1-\beta(1-h)}$. Observe that Φ and Δ are continuous functions of h, that Φ increases with h, while Δ decreases with h and that $\Phi(h=0)=0$ and $\Delta(h=0) = \frac{\beta}{1-\beta}$.

A.1 Proof of Theorem 1

Proof.

We classify stationary strategy profiles into three types: *no-compromise (or interior)* if $w_0, w_1 > 0$; *partial-compromise* if ($w_0 \leq 0$ and $w_1 > 0$) or ($w_0 > 0$ and $w_1 \leq 0$); and *full-compromise* if $w_0, w_1 \leq 0$. In the sequel, we show that for h small enough, only interior equilibria exist.

1. Reservation values We start by computing parties' reservation values as functions of the agreement set (at a given stationary profile). We apply (1), by noting $\lambda := \mathbb{P}(x \in A)$ and observing that, since $u_1(x) = x+1$, we have $\mathbb{E}[u_1(x) \mid x \in A] = u_1(c) = 1+c$. We obtain the reservation value for party 1:

$$w_1 = \frac{hd_1 + \beta(1-h)\lambda(1+c)}{1-\beta(1-h)+\beta(1-h)\lambda} = \frac{\Phi d_1 + \Delta\lambda(1+c)}{1+\Delta\lambda}. \tag{3}$$

Similarly, as $u_0(x) = 1-x$, we have $\mathbb{E}[u_0(x) \mid x \in A] = u_0(c) = 1-c$. By application of (1), we obtain:

$$w_0 = \frac{\Phi d_0 + \Delta\lambda(1-c)}{1+\Delta\lambda}.$$

2. Agreement sets in equilibrium

No-compromise equilibrium. Let w be a no-compromise equilibrium, i.e. such that $w_0 > 0$ and $w_1 > 0$. The agreement set is thus $A = [c-\lambda, c+\lambda] = [-1+w_1, 1-w_0]$. We obtain

$$\begin{cases} w_1 + w_0 & = 2(1-\lambda) \\ w_1 - w_0 & = 2c. \end{cases}$$

Solving for c first, we get:

$$2c = \frac{\Phi(d_1 - d_0) + \Delta\lambda 2c}{1 + \Delta\lambda} \quad \Leftrightarrow \quad c = \frac{\Phi(d_1 - d_0)}{2} = \frac{\Phi B}{2}. \tag{4}$$

Solving for λ, we obtain :

$$2(1-\lambda) = \frac{\Phi(d_0 + d_1) + 2\Delta\lambda}{1 + \Delta\lambda} = \frac{\Phi D + 2\Delta\lambda}{1 + \Delta\lambda} \quad \Leftrightarrow \quad 1 - \frac{\Phi D}{2} = \lambda + \Delta\lambda^2. \tag{5}$$

Hence, we get:

$$\lambda = \frac{1}{2\Delta}\left(\sqrt{1 + 4\Delta\left(1 - \frac{\Phi D}{2}\right)} - 1\right). \tag{6}$$

Partial-compromise equilibrium ($w_0 \leq 0$ and $w_1 > 0$). Let w be a partial-compromise equilibrium with $w_0 \leq 0$ and $w_1 > 0$. Then we have $A = [-1 + w_1, 1]$, so that some proposals that are too far from party 1's bliss point are rejected. We have $w_1 = 2(1-\lambda)$ and $1 + c = 1 + \frac{w_1}{2} = 2(1 - \frac{\lambda}{2})$. Using (3), we obtain:

$$2(1-\lambda)(1+\Delta\lambda) = \Phi d_1 + 2\Delta\lambda\left(1 - \frac{\lambda}{2}\right) \quad \Leftrightarrow \quad 1 - \frac{\Phi d_1}{2} = \lambda + \Delta\frac{\lambda^2}{2}.$$

Hence, we get:

$$\lambda = \frac{1}{\Delta}\left(\sqrt{1 + 2\Delta\left(1 - \frac{\Phi d_1}{2}\right)} - 1\right).$$

The center of the agreement set is given by $c = 1 - \lambda$.

Full-compromise equilibrium. In a full-compromise equilibrium ($w_0 \leq 0$ and $w_1 \leq 0$), the agreement set is $A = X = [-1, 1]$.

3. Conditions for existence

We now show that: a no-compromise equilibrium exists for h small and only this sort of stationary equilibrium exists (uniqueness then follows from the formulas derived above).

No-compromise equilibrium: existence. A no-compromise equilibrium is characterized by the system of equations: $c = \frac{\Phi B}{2}$ and $1 - \frac{\Phi D}{2} = \lambda + \Delta\lambda^2$. As $c = \frac{\Phi B}{2} \geq 0$, a necessary and sufficient condition for such an equilibrium to exist is that the previous system admits a solution with $c + \lambda \leq 1$, or equivalently $\lambda \leq 1 - \frac{\Phi B}{2}$. Hence, a no-compromise equilibrium exists if and only if:

$$\exists \lambda < 1 - \frac{\Phi B}{2}, \qquad 1 - \frac{\Phi D}{2} = \lambda + \Delta\lambda^2.$$

Now, observe that $\Phi(h = 0) = 0$, that $\Delta(h = 0) = \frac{\beta}{1-\beta} > 0$ and that Φ and Δ are continuous functions of h. It follows that the previous condition is satisfied (and thus that a no-compromise equilibrium exists) for h small enough.

Full-compromise equilibrium: existence. In a full-compromise equilibrium, we have $c = 0$ and $\lambda = 1$. As $B \geq 0$, we have $w_1 \geq w_0$, and a necessary and sufficient condition for existence is then $w_1 \leq 0$. This can be written $w_1 = \frac{\Phi d_1 + \Delta}{1+\Delta} \leq 0$, or equivalently $\Phi d_1 + \Delta \leq 0$. As Φ and Δ are continuous at $h = 0$, with $\Phi(h = 0) = 0$, $\Delta(h = 0) > 0$ and $d_1 < 0$, a full-compromise equilibrium does not exist for h small enough.

Partial-compromise equilibrium: existence. Let us first consider the case $w_0 \leq 0$ and $w_1 > 0$. As shown above, a partial-compromise equilibrium is characterized by the equation $1 - \frac{\Phi d_1}{2} = \lambda + \Delta\frac{\lambda^2}{2}$, or equivalently $1 - \frac{\Phi(B+D)}{4} = \lambda + \Delta\frac{\lambda^2}{2}$. Such an equilibrium exists if and only if $w_0 \leq 0$ and $\lambda \leq 1$. The lower bound of the agreement set is $-1 + w_1 = 1 - 2\lambda$, it follows that $w_1 = 2(1 - \lambda)$. We may thus write:

$$\begin{aligned} w_0 = (w_0 + w_1) - w_1 &= \frac{\Phi(d_0 + d_1) + \Delta\lambda[(1-c) + (1+c)]}{1 + \Delta\lambda} - 2(1-\lambda) \\ &= \frac{\Phi D + 2\Delta\lambda - 2(1-\lambda)(1+\Delta\lambda)}{1+\Delta\lambda}. \end{aligned}$$

Hence, we can write :

$$w_0 \leq 0 \;\Leftrightarrow\; \frac{\Phi D}{2} + \Delta\lambda \leq (1-\lambda)(1+\Delta\lambda) \Leftrightarrow\; \lambda + \Delta\lambda^2 \leq 1 - \frac{\Phi D}{2}$$
$$\Leftrightarrow\; 2\left(\lambda + \Delta\frac{\lambda^2}{2}\right) - \lambda \leq 1 - \frac{\Phi D}{2}$$
$$\Leftrightarrow\; 2\left(1 - \frac{\Phi(B+D)}{4}\right) - \lambda \leq 1 - \frac{\Phi D}{2}$$
$$\Leftrightarrow\; 1 - \frac{\Phi B}{2} \leq \lambda.$$

To conclude, such a partial-compromise equilibrium exists if and only if:

$$\exists \lambda \in [1 - \frac{\Phi B}{2}, 1), \qquad 1 - \frac{\Phi(B+D)}{4} = \lambda + \Delta\frac{\lambda^2}{2}.$$

As Φ and Δ are continuous at $h = 0$, with $\Phi(h = 0) = 0$, $\Delta(h = 0) > 0$, such a partial-compromise equilibrium does not exist for h small enough.

Consider now the case $w_0 > 0$ and $w_1 \leq 0$. We obtain as above (replacing d_1 by d_0) that $1 - \frac{\Phi(D-B)}{4} = \lambda + \Delta\frac{\lambda^2}{2}$. Following the same steps as above, we also obtain that

$$w_1 \leq 0 \quad\Leftrightarrow\quad 2\left(1 - \frac{\Phi(D-B)}{4}\right) - \lambda \leq 1 - \frac{\Phi D}{2} \quad\Leftrightarrow\quad 1 + \frac{\Phi B}{2} \leq \lambda.$$

This last condition is incompatible with $\lambda < 1$, which must hold at a partial-compromise equilibrium. Hence, such a partial-compromise equilibrium cannot exist.

To conclude, for h small enough : a no-compromise equilibrium exists, it is unique, and there are no other stationary equilibria.

4. Comparative statics.

Since Φ is increasing in h, it is immediate that $c = \frac{\Phi B}{2}$ is a non-decreasing function of h, strictly increasing whenever $B > 0$.

The agreement probability λ is obtained as the solution of the equation $1 - \frac{\Phi D}{2} = \lambda + \Delta\lambda^2$. As shown on Figure 6, λ increases as h increases.

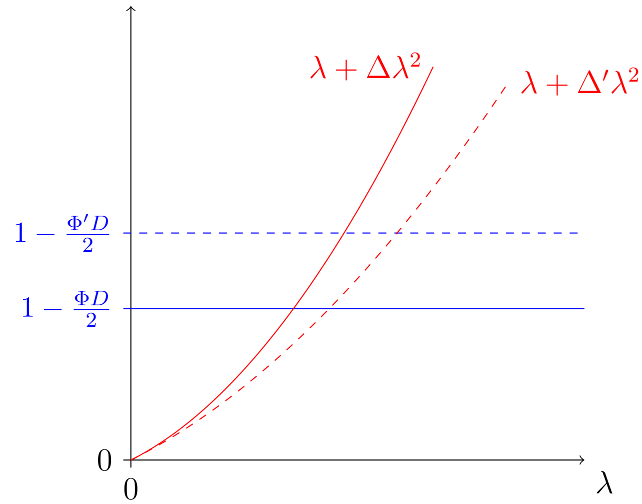

Figure 6: Characterization of λ (h increases)

■

A.2 Proof of Proposition 3

Proof. Let w be an equilibrium and let A be its agreement set. We apply (1) and (2):

$$\begin{aligned} W_\theta &= \frac{(1-\lambda)\, hd_\theta + \lambda \mathbb{E}[u_\theta(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)}, \\ w_\theta &= \frac{hd_\theta + \beta(1-h)\lambda \mathbb{E}[u_\theta(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)}. \end{aligned}$$

Solving for $\lambda \mathbb{E}[u_\theta(x) \mid x \in A]$ in both expressions, we have:

$$\begin{aligned} \frac{\lambda \mathbb{E}[u_\theta(x) \mid x \in A]}{1 - \beta(1-h)\,(1-\lambda)} &= W_\theta - \frac{(1-\lambda)\, h}{1 - \beta(1-h)\,(1-\lambda)} d_\theta \\ &= \frac{1}{\beta(1-h)} \left(w_\theta - \frac{h}{1 - \beta(1-h)\,(1-\lambda)} d_\theta \right). \end{aligned}$$

Thus, simplifying, we have the affine relation:

$$W_\theta = \frac{1}{\beta(1-h)} \left(w_\theta - hd_\theta \right). \tag{7}$$

Since w is an interior equilibrium, we have $w_0 + w_1 = 2(1-\lambda)$, and we may write :

$$W = \frac{1-\lambda-\frac{hD}{2}}{\beta(1-h)}.$$

Applying (5), we obtain:

$$\begin{aligned}
1-\frac{\Phi D}{2} = \lambda + \Delta\lambda^2 \quad &\Leftrightarrow \quad 1-\frac{h}{1-\beta(1-h)} \times \frac{D}{2} = \lambda + \frac{\beta(1-h)}{1-\beta(1-h)}\lambda^2 \\
&\Leftrightarrow \quad 1-\beta(1-h)-\frac{hD}{2} = \left(1-\beta(1-h)\right)\lambda + \beta(1-h)\lambda^2 \\
&\Leftrightarrow \quad 1-\lambda-\frac{hD}{2} = \beta(1-h)\left(1-\lambda+\lambda^2\right).
\end{aligned}$$

We thus obtain:

$$W = \frac{1-\lambda-\frac{hD}{2}}{\beta(1-h)} = \frac{1-\lambda-\frac{hD}{2}}{\beta(1-h)} = 1-\lambda+\lambda^2,$$

as desired. ■

A.3 Proof of Proposition 4

Proof. Applying (2) and noting $f(h) = \dfrac{1-(1-h)(1-\lambda)}{1-\beta(1-h)(1-\lambda)}$, we obtain:

$$W = \frac{\lambda+(1-\lambda)h\frac{D}{2}}{1-\beta(1-h)(1-\lambda)} = \frac{\lambda+(1-\lambda)h\frac{D}{2}}{1-(1-h)(1-\lambda)} \times f(h).$$

Moreover, we may write:

$$\begin{aligned}
\frac{W}{f(h)} &= \frac{\lambda+(1-\lambda)h\frac{D}{2}}{1-(1-h)(1-\lambda)} \\
&= 1+\left(\frac{\lambda}{1-(1-h)(1-\lambda)}-1\right)+\frac{(1-\lambda)h\frac{D}{2}}{1-(1-h)(1-\lambda)} \\
&= 1+\frac{\lambda-1+(1-h)(1-\lambda)}{1-(1-h)(1-\lambda)}+\frac{(1-\lambda)h\frac{D}{2}}{1-(1-h)(1-\lambda)} \\
&= 1+\left(\frac{D}{2}-1\right)\frac{(1-\lambda)h}{1-(1-h)(1-\lambda)}.
\end{aligned}$$

To conclude, observe first that:

$$\mathbb{P}(d^* \mid A) = (1-\lambda)h \sum_{k=0}^{+\infty} (1-\lambda)^k (1-h)^k = \frac{(1-\lambda)h}{1-(1-\lambda)(1-h)}.$$

Second, observe that $(1-h)(1-\lambda)$ decreases with h, while $\frac{1-x}{1-\beta x}$ decreases with x, as $\beta < 1$. Together, this implies that the timing factor $f(h)$ is indeed increasing in h. ■

A.4 Proof of Theorem 2

Proof. 1. Formally, we show in this proof that: for any $B > 0$, there exists $\overline{\beta} \in (0,1)$ such that for any $\beta \geq \overline{\beta}$, we have $\dfrac{\partial W_1}{\partial h}(h=0) > 0$.

Let w be an equilibrium and let A be its agreement set. We know from Proposition 3 that $W = 1 - \lambda + \lambda^2$. Since w is interior, we have $c = \dfrac{(-1+w_1)+(1-w_0)}{2}$, so that, using (4), we obtain $w_1 - w_0 = 2c = \Phi B$. Using (7), we may write:

$$\begin{aligned} W_1 - W_0 &= \frac{1}{\beta(1-h)} \left(w_1 - w_0 - h(d_1 - d_0)\right) \\ &= \frac{1}{\beta(1-h)} \left(\Phi B - hB\right) = \frac{hB}{\beta(1-h)} \left(\frac{1}{1-\beta(1-h)} - 1 \right) = \Phi B. \end{aligned}$$

Thus, we obtain

$$W_1 = \frac{W_0 + W_1}{2} + \frac{W_1 - W_0}{2} = 1 - \lambda + \lambda^2 + \frac{B\Phi}{2}.$$

We thus have $\frac{\partial W_1}{\partial h} = (2\lambda - 1)\frac{\partial \lambda}{\partial h} + \frac{B}{2}\frac{\partial \Phi}{\partial h}$. Let us study the two terms separately.

Noting $\delta = 1 - \beta$, we have that $\frac{\partial \Phi}{\partial h}(h=0) = \frac{1}{1-\beta} = \frac{1}{\delta}$.

Differentiating (5) with respect to h, we obtain:

$$0 = \frac{D}{2}\frac{\partial \Phi}{\partial h} + \lambda^2 \frac{\partial \Delta}{\partial h} + \frac{\partial \lambda}{\partial h}(1 + 2\Delta\lambda).$$

As $\frac{\partial \Delta}{\partial h}(h=0) = \frac{-\beta}{(1-\beta)^2} = \frac{-(1-\delta)}{\delta^2}$ and $\Delta(h=0) = \frac{\beta}{1-\beta} = \frac{1-\delta}{\delta}$, we may write:

$$\frac{\partial \lambda}{\partial h}(h=0) = \frac{-\frac{D}{2\delta} + \frac{(1-\delta)\lambda_0^2}{\delta^2}}{1 + \frac{2\lambda_0(1-\delta)}{\delta}} = \frac{-\frac{D\delta}{2} + (1-\delta)\lambda_0^2}{\delta^2 + 2\lambda_0(1-\delta)\delta} \quad \text{with} \quad \lambda_0 = \lambda(h=0).$$

Applying (6), we obtain:

$$\lambda_0 = \frac{1}{2\Delta_0}(\sqrt{1+4\Delta_0} - 1) = \frac{1 + 4\Delta_0 - 1}{2\Delta_0(\sqrt{1+4\Delta_0} + 1)} = \frac{2}{\sqrt{1+4\Delta_0} + 1},$$

with $\Delta_0 = \Delta(h=0)$. For small δ, we obtain that:

$$\lambda_0 = \frac{2}{1 + \sqrt{1 + 4\frac{1-\delta}{\delta}}} = \frac{2}{\sqrt{\frac{1}{\delta}}(\sqrt{\delta} + \sqrt{4-3\delta})} \sim \sqrt{\delta}.$$

We thus have, for small δ:

$$\frac{\partial \lambda}{\partial h}(h=0) \sim \frac{-\frac{D\delta}{2} + (1-\delta)\delta}{\delta^2 + 2\sqrt{\delta}(1-\delta)\delta} \sim \frac{-\frac{D}{2} + (1-\delta)}{2\sqrt{\delta}(1-\delta)} \sim \frac{1 - \frac{D}{2}}{2\sqrt{\delta}}.$$

To sum up, the welfare W_1 of the advantaged party satisfies, for small δ, the following condition:

$$\frac{\partial W_1}{\partial h}(h=0) = (2\lambda_0 - 1)\frac{\partial \lambda}{\partial h}(h=0) + \frac{B}{2}\frac{\partial \Phi}{\partial h}(h=0) \sim -\frac{1 - \frac{D}{2}}{2\sqrt{\delta}} + \frac{B}{2\delta} \sim \frac{B}{2\delta}.$$

Hence, for $B > 0$ we obtain that $\frac{\partial W_1}{\partial h}(h=0) > 0$ for δ small enough, that is, for β close enough to 1.

2. When $D = -B$, party 1's utility derivative at $h = 0$ writes

$$\begin{aligned}
\frac{\partial W_1}{\partial h}(h=0) &= (2\lambda_0 - 1)\frac{\partial \lambda}{\partial h}(h=0) + \frac{B}{2}\frac{\partial \Phi}{\partial h}(h=0) \\
&= (2\lambda_0 - 1)\frac{\beta\lambda_0^2 + \frac{B(1-\beta)}{2}}{(1-\beta)^2 + 2\beta(1-\beta)\lambda_0} + \frac{B}{2(1-\beta)} \\
&= \frac{2(2\lambda_0 - 1)\beta\lambda_0^2 + B(1-\beta)(2\lambda_0 - 1) + B(1-\beta) + 2B\beta\lambda_0}{2(1-\beta)^2 + 4\beta(1-\beta)\lambda_0} \\
&= \frac{2(2\lambda_0 - 1)\beta\lambda_0^2 + 2B\lambda_0}{2(1-\beta)^2 + 4\beta(1-\beta)\lambda_0} \\
&= \frac{\lambda_0}{(1-\beta)^2 + 2\beta(1-\beta)\lambda_0}\left(\beta\lambda_0(2\lambda_0 - 1) + B\right).
\end{aligned}$$

Hence:

- for any $\beta < 2/3$, we have $\lambda_0 < 1/2$ and thus $\frac{\partial W_1}{\partial h}(h=0) > 0$.
- for any $\beta \geq 2/3$, we have $\frac{\partial W_1}{\partial h}(h=0) > 0$ whenever $B > \beta\lambda_0|2\lambda_0 - 1|$.

Hence, a necessary and sufficient condition to have $\frac{\partial W_1}{\partial h}(h=0) > 0$ when $D = -B$ for any β is

$$B > \min_{\beta \geq 2/3} \beta\lambda_0|2\lambda_0 - 1| := \overline{B}$$

As $\lambda|2\lambda - 1] \leq \frac{1}{8}$, we obtain that $\overline{B} < \frac{1}{8}$.[15]

To conclude, we have shown that for any $D < -\frac{1}{8}$, party 1 has an incentive to activate the brinkmanship strategy if she is advantaged enough (if $B = -D$, and by continuity, if $B < -D$ is high enough), independently on the discount factor β. ■

[15] We use this upper bound on $\overline{B}$ for simplicity in the statement of the result. Numerically, we find that $\overline{B} \approx 0.116$.

A.5 Proof of Proposition 5

Proof. Using the same notations as in the proof of Theorem 2, we have that $\lambda_0 > 1/2$ when $\beta < 2/3$. As $W_0 = 1 - \lambda + \lambda^2 - \frac{B\Phi}{2}$, party 0's utility derivative at $h = 0$ then writes:

$$\begin{aligned}
\frac{\partial W_0}{\partial h}(h=0) &= (2\lambda_0 - 1)\frac{\partial \lambda}{\partial h}(h=0) - \frac{B}{2}\frac{\partial \Phi}{\partial h}(h=0) \\
&= (2\lambda_0 - 1)\frac{\beta\lambda_0^2 - \frac{D(1-\beta)}{2}}{(1-\beta)^2 + 2\beta(1-\beta)\lambda_0} - \frac{B}{2(1-\beta)} \\
&\geq (2\lambda_0 - 1)\frac{\beta\lambda_0^2 + \frac{B(1-\beta)}{2}}{(1-\beta)^2 + 2\beta(1-\beta)\lambda_0} - \frac{B}{2(1-\beta)}.
\end{aligned}$$

This fraction is positive for $B = 0$ (and thus by continuity for B small enough), hence the result. ■

References

Avidit Acharya and Edoardo Grillo. War with crazy types. *Political Science Research and Methods*, 3(2):281–307, 2015.

Avidit Acharya and Juan Ortner. Paths to the frontier. *American Economic Journal: Microeconomics*, 14(1):39–69, 2022.

James Albrecht, Axel Anderson, and Susan Vroman. Search by committee. *Journal of Economic Theory*, 145(4):1386–1407, 2010.

Deepal Basak and Joyee Deb. Gambling over public opinion. *American Economic Review*, 110(11):3492–3521, 2020.

Ken Binmore, Ariel Rubinstein, and Asher Wolinsky. The nash bargaining solution in economic modelling. *The RAND Journal of Economics*, pages 176–188, 1986.

T. Renee Bowen, Ying Chen, Hülya Eraslan, and Jan Zápal. Efficiency of flexible budgetary institutions. *Journal of Economic Theory*, 167:148–176, 2017.

Shirish D. Chikte and Sudhakar D. Deshmukh. The role of external search in bilateral bargaining. *Operations Research*, 35(2):198–205, 1987.

Olivier Compte and Philippe Jehiel. Bargaining and majority rules: A collective search perspective. *Journal of Political Economy*, 118(2):189–221, 2010.

Olivier Compte and Philippe Jehiel. On the optimal majority rule. Technical report, 2017.

Peter C. Cramton and Joseph S. Tracy. Strikes and holdouts in wage bargaining: theory and data. *The American Economic Review*, 82:100–121, 1992.

Bhaskar Dutta and Arunava Sen. Nash implementation with partially honest individuals. *Games and Economic Behavior*, 74(1):154–169, 2012.

Wioletta Dziuda and Antoine Loeper. Dynamic collective choice with endogenous status quo. *Journal of Political Economy*, 124(4):1148–1186, 2016.

Tore Ellingsen and Topi Miettinen. Commitment and conflict in bilateral bargaining. *American Economic Review*, 98(4):1629–35, 2008.

Hülya Eraslan and Antonio Merlo. Majority rule in a stochastic model of bargaining. *Journal of Economic Theory*, 103(1):31–48, 2002.

William Fuchs and Andrzej Skrzypacz. Bargaining with deadlines and private information. *American Economic Journal: Microeconomics*, 5(4):219–43, 2013.

Uri Gneezy, Ernan Haruvy, and Alvin E Roth. Bargaining under a deadline: Evidence from the reverse ultimatum game. *Games and Economic Behavior*, 45(2):347–368, 2003.

Edoardo Grillo and Carlo Prato. Reference points and democratic backsliding. *American Journal of Political Science*, 2020.

Albert Ma and Michael Manove. Bargaining with deadlines and imperfect player control. *Econometrica*, 61(6):1313–1339, 1993.

Paola Manzini and Marco Mariotti. Going alone together: joint outside options in bilateral negotiations. *The Economic Journal*, 114(498):943–960, 2004.

Nolan McCarty. *Polarization: What everyone needs to know®*. Oxford University Press, 2019.

Antonio Merlo and Charles Wilson. A stochastic model of sequential bargaining with complete information. *Econometrica*, 63(2):371–399, 1995.

Antonio Merlo and Charles Wilson. Efficient delays in a stochastic model of bargaining. *Economic Theory*, 11(1):39–55, 1998.

Matthias Messner and Mattias K. Polborn. The option to wait in collective decisions and optimal majority rules. *Journal of Public Economics*, 96(5-6):524–540, 2012.

Benny Moldovanu and Frank Rosar. Brexit: A comparison of dynamic voting games with irreversible options. *Games and Economic Behavior*, 130:85–108, 2021.

Benny Moldovanu and Xianwen Shi. Specialization and partisanship in committee search. *Theoretical Economics*, 8(3):751–774, 2013.

Abhinay Muthoo. On the strategic role of outside options in bilateral bargaining. *Operations Research*, 43(2):292–297, 1995.

Juan Ortner. A theory of political gridlock. *Theoretical Economics*, 12(2):555–586, 2017.

John W. Patty. Signaling through obstruction. *American Journal of Political Science*, 60 (1):175–189, 2016.

Elizabeth Maggie Penn. A model of farsighted voting. *American Journal of Political Science*, 53(1):36–54, 2009.

Robert Powell. Nuclear brinkmanship with two-sided incomplete information. *American Political Science Review*, 82(1):155–178, 1988.

Michael Schwarz and Konstantin Sonin. A theory of brinkmanship, conflicts, and commitments. *The Journal of Law, Economics, & Organization*, 24(1):163–183, 2008.

Alp Simsek and Muhamet Yildiz. Durability, deadline, and election effects in bargaining. Technical report, National Bureau of Economic Research, 2016.

Bruno Strulovici. Learning while voting: Determinants of collective experimentation. *Econometrica*, 78(3):933–971, 2010.

Asher Wolinsky. Matching, search, and bargaining. *Journal of Economic Theory*, 42(2): 311–333, 1987.

Janet Yellen. Secretary Yellen sends debt limit letter to congress (1/13/2023). *U.S. Department of the Treasury (available at https://home.treasury.gov/system/files/136/Debt-Limit-Letter-to-Congress-McCarthy-20230113.pdf)*, 2023.